OPTIMISATION

In practical solutions, finding the optimum value is finding a minimum or maximum value.

A reminder of the process to find a minimum or maximum:

Given a function, e.g. $y = x^3 + 8x^2$, we find the minimum or maximum value using the following steps:

1. Differentiate the function to find $f'(x)$ — e.g. $y' = 3x^2 + 16x$
2. Put $f'(x)$ equal to 0, as gradient at stationary points is zero — e.g. $3x^2 + 16x = 0$
3. Solve this eqn to find the x-value of the stationary points — e.g. $x(3x + 16) = 0$
   therefore $x = 0$ or $x = -5\frac{1}{3}$
4. Substitute these values into the original function to find the corresponding y-values of the stationary points — e.g. when $x = 0, y = 0$
   and when $x = -5\frac{1}{3}, y = 75.851$

There is therefore a minimum when $x = 0$ and a maximum when $= -5\frac{1}{3}$. This can be established by using the Second Derivative Test:

- If $\frac{d^2y}{dx^2} < 0$ then the turning point is a maximum
- If $\frac{d^2y}{dx^2} > 0$ then the turning point is a minimum

but at Achieved level you will not be required to determine this.

Example:

The total profit when manufacturing and selling x children's ski jackets is given by the equation: $(x) = -0.04x^2 + 16x - 80$. Find the number of ski-jackets which will maximise the profit, and find this profit.

You may assume that $P''(x) < 0$. The last statement above lets you know that your result will be a maximum.

Step 1: Differentiating $P(x)$: $P'(x) = -0.08x + 16$
Step 2: Let $P'(x) = 0$: $-0.08x + 16 = 0$
Step 3: Solve $-0.08x + 16 = 0 \quad => x = 200$
Step 4: Substitute When $x = 200, P(x) = \$1520$

Therefore, the maximum profit will occur when 200 jackets are sold, and that profit will be \$1520.

Worksheet